17-07-14

UNIT - II

FLOW THROUGH CIRCULAR CONDUITS

Hydraulic and energy gradient - Laminar flow through circular conduits and circular annuli - Boundary layer concept - Types of boundary layer thickness - Darcy Weisbach equation - friction factor - Moody diagram - commercial pipes - minor losses - Flow through pipe in series and parallel.

P.NO: 387

Flow of viscous fluid through circular pipe :

The flow of viscous fluid through circular pipe, the velocity distribution across a section, the ratio of maximum velocity to average velocity, the shear stress distribution and drop of Pressure for a given length is to be determined.

$$Re = \frac{\rho V D}{\mu}$$

$\bar{u} = v$ → equal.

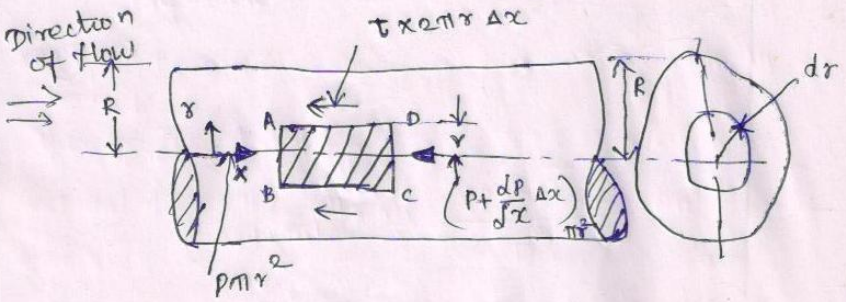
where,

ρ = Density of fluid flowing through pipe

V = Average velocity of fluid

D = Diameter of pipe

μ = viscosity of fluid.



Less than 2000 ⟹ Laminar flow

More than 4000 → Turbulant flow

↓

Reynold number

(x)

Hagen Poiseuille Formula :

$$(P_1 - P_2) = \frac{32 \mu \bar{u} L}{D^2}$$

where,

$P_1 - P_2$ is the drop of Pressure

∴ Loss of Pressure head $= \frac{P_1 - P_2}{\rho g}$

$$\therefore \frac{P_1 - P_2}{\rho g} = h_f = \frac{32 \mu \bar{u} L}{\rho g D^2}$$

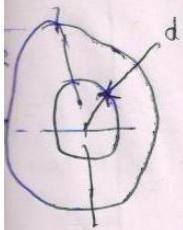
Hydraullic Gradient & Total Energy line :-

Hydraulic Gradient line :

It is defined as the line which gives the sum of Pressure head $\left(\frac{P}{w}\right)$ and datum head (z).

Total Energy Line :

It is defined as the line which gives the sum of pressure head, datum head & kinetic head.



1) An oil of viscosity $0.1\ NS/m^2$ and relative density 0.9 is flowing though a circular pipe of diameter 50 mm and length of 3000 m. The rate of flow of fluid through the pipe is $3.5\ lit/sec$. Find the Pressure drop in a length of 3000m.

Data:

Viscosity $\mu = 0.1\ NS/m^2$

Relative density $= 0.9$ $\quad \rho_{relative} = \frac{\rho_{oil}}{\rho_{water}}$

Density of oil $= 0.9 \times 1000$ $\quad \rho_{oil} = \rho_{water} \times \rho_{relative}$

$= 900\ kg/m^3$ $\quad = 1000 \times 0.9$

$\rho_{oil} = 900\ kg/m^3$

Diameter of pipe, $D = 50\ mm \Rightarrow 0.05\ m$

$L = 3000\ m$

$Q = 3.5\ lit/s$

$\Rightarrow 0.0035\ m^3/s$

Find:

i) Pressure drop $(P_1 - P_2)$

Sol:

Pressure drop,

$$(P_1 - P_2) = \frac{32 \mu \bar{V} L}{D^2}$$

Area, $A = \pi/4\ D^2$

$= \pi/4 \times (0.05)^2$

$A = 1.96 \times 10^{-3}\ m^2$

~~$A = 0.1963\ m^2$~~ ~~$A = 0.00$~~

$$Q = A \times V$$

$$V = \frac{Q}{A} = \frac{0.0035}{1.96 \times 10^{-3}} = 1.78 \text{ m/s}$$

$$V = \bar{U} = 1.78 \text{ m/s}$$

Pressure drop,

$$(P_1 - P_2) = \frac{32 \mu \bar{U} L}{D^2}$$

$$= \frac{32 \times 0.1 \times 1.78 \times 3000}{(0.05)^2}$$

$$(P_1 - P_2) = 6835200 \text{ N/m}^2$$

$$\boxed{(P_1 - P_2) = 68.35 \text{ N/cm}^2}$$

P.NO. 465

CHAPTER - 11

FLOW THROUGH PIPES

Laminar Flow :

Reynolds number is less than 2000 for pipe flow, the flow known as "laminar flow".

Turbulant Flow :

Reynolds number is more than 4000 for pipe flow, the flow known as "turbulant flow".

Loss of Energy in pipes :

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost.

Energy losses

- **Major Energy** → This is due to friction calculated by the following formulae,
  - a) Darcy - weisbach formula
  - b) Chezy's formulae
- **Minor energy** →
  - a) Sudden expansion of Pipe
  - b) Sudden contraction of Pipe
  - c) Bend in pipe
  - d) Pipe fittings etc.
  - e) An obstruction in pipe

loss

flow

more

e flow

g through

some

of the

energy

xpansion of Pipe

ntraction of Pipe

pipe

ings etc.

ruction in Pipe

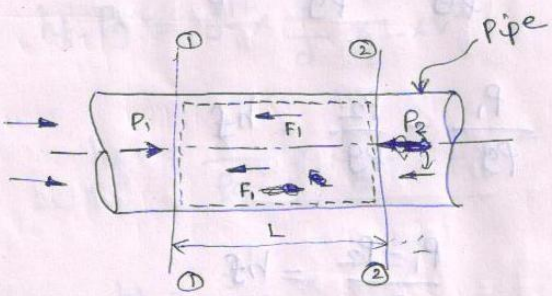
Page No: 434

Darcy-Weisbach Equation :

$P_1 \longrightarrow (+)$

$P_2 \longleftarrow (-)$

16 mark → 434 - 436



Let,

P_1 = Pressure intensity at section 1-1,

V_1 = Velocity of flow at section 1-1,

L = length of the pipe b/w sections 1-1 and 2-2

d = diameter of Pipe

f' = frictional resistance per unit wetted area per unit velocity

h_f = loss of head due to friction

P_2 = Pressure intensity at section 2-2

V_2 = velocity of flow at section 2-2

Energy equation,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$$

Pipe in horizontal, $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

Pipe is uniform velocity $\therefore V_1 = V_2$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = h_f$$

$$\frac{P_1 - P_2}{\rho g} = h_f$$

$$\boxed{P_1 - P_2 = h_f \cdot \rho g}$$

Frictional resistance } = frictional resistance per unit wetted area per unit velocity $\times$ (Velocity)^2

$$F_1 = f' \times \pi d L \times V^2 \quad \left[\because \text{wetted area} = \pi d \times L, \text{ velocity} = V = V_1 = V_2\right]$$

$$F_1 = f' \times P \times L \times V^2 \quad [\pi d = \text{Perimeter} = P]$$

$$P_1 A - P_2 A - F = 0$$

$$(P_1 - P_2) A - F = 0$$

$$(P_1 - P_2) A = F$$

$$P_1 - P_2 = \frac{F}{A}$$

$$P_1 - P_2 = \frac{f' \times P \times L \times V^2}{A}$$

$$h_f \cdot \rho g = \frac{f' \times P \times L \times V^2}{A}$$

$$\frac{P}{A} = \frac{\pi d}{\pi d^2/4} = \frac{4}{d}$$

$$\Rightarrow \quad h_f \cdot \rho g = f' \times \frac{4}{d} \times L \times V^2$$

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2$$

$$h_f = \frac{f'}{\rho g} \times \frac{4LV^2}{d}$$

Putting, $\frac{f'}{\rho} = \frac{f}{2}$

where, $f \rightarrow$ is known as co-efficient of friction

$$h_f = \frac{f}{2g} \times \frac{4LV^2}{d}$$

$$\boxed{h_f = \frac{4fLV^2}{2gd}} \rightarrow \text{Darcy - equation}$$

f - find formulae:

$$f = \frac{16}{Re}$$

for $Re < 2000$ (viscous flow)

$$f = \frac{0.079}{Re^{1/4}}$$ (varying from) 4000 to 10^6

$f \rightarrow$ co-effiuent of friction

$$Re = \frac{\overrightarrow{v} \times d}{\vartheta}$$ ($\overrightarrow{v}$ → velocity, ϑ → viscosity)

P.NO. 466

Chezy's Formulae for loss of head due to friction in pipes:

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times v^2$$

$\frac{A}{P} = \frac{\text{Area of flow}}{\text{Perimeter}}$ is called "Hydraulic mean depth (or) hydraulic radius and is denoted by "m",

Hydraulic mean depth, $m = \frac{A}{P}$

$$m = \frac{\pi d^2/4}{\pi d} = \frac{d}{4}$$

$$\frac{A}{P} = m \quad (\text{or}) \quad \frac{P}{A} = \frac{1}{m}$$

$$\Rightarrow h_f = \frac{f'}{\rho g} \times \frac{1}{m} \times L \times v^2$$

$$v^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L}$$

$$v = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}}$$

$$\sqrt{\frac{\rho g}{f'}} = C \qquad \therefore \boxed{v = C\sqrt{mi}}$$

where, C = constant

$$\frac{h_f}{L} = i$$

where, i = loss of head per unit length of pipe

Problems:

1) Find the head lost due to friction in a pipe of diameter 300 mm & length 50 m, through which water is flowing at a velocity of 3 m/s using i) Darcy formula, ii) Chezy's formula for which $c = 60$.

Take ν for water $= 0.01$ stoke

Data:

Dia. of Pipe, $d = 300$ mm $\Rightarrow \frac{300}{1000} = 0.30$ m

length of Pipe, $L = 50$ m

velocity of flow, $v = 3$ m/s

chezy's constant, $c = 60$

kinematic viscosity, $\nu = 0.01$ stoke

$\Rightarrow 0.01$ cm^2/s

$\nu \Rightarrow 0.01 \times 10^{-4}$ m^2/s

Find:

i) Darcy formula

ii) chezy's formula

sol:

i) Darcy Formulae,

$$h_f = \frac{4.f.L.v^2}{2gd}$$

$$f = Re = \frac{\text{velocity} \times \text{diameter}}{\text{viscosity}}$$

$$= \frac{3 \times 0.30}{0.01 \times 10^{-4}}$$

$$Re = 9 \times 10^5$$

Re → is more than 4000, so its formula

$$f = \frac{0.079}{Re^{1/4}}$$

$$= \frac{0.079}{(9\times10^5)^{1/4}}$$

$$f = 0.00256$$

∴ Head lost, $h_f = \frac{4f.LV^2}{2gd}$

$$= \frac{4\times0.00256\times50\times(3)^2}{2\times9.81\times0.30}$$

$$\boxed{h_f = 0.7825\ m}$$

ii) chezy formulae,

$$V = C\sqrt{mi}$$

$$m = \frac{d}{4} = \frac{0.30}{4} = 0.075\ m$$

$$3 = 60\sqrt{0.075\times i}$$

$$\frac{3}{60} = \sqrt{0.075\times i}$$

$$0.05 = \sqrt{0.075}\times\sqrt{i}$$

$$\frac{0.05}{\sqrt{0.075}} = \sqrt{i}$$

$$0.1825 = \sqrt{i}$$

$$(0.1825)^2 = (\sqrt{i})^2$$

$i^{\circ} = 0.03333$

$$i = \frac{h_f}{L}$$

$$h_f = i \times L$$

$$= 0.0333 \times 50$$

$$\boxed{h_f = 1.665 \text{ m}}$$

2) Find the diameter of a pipe of length 2000 m when the rate of flow of water through the pipe is 200 litres/s and the head lost due to friction is 4 m. Take the value of $C = 50$ in chezy's formulae.

Data:

Length, $L = 2000$ m

Discharge, $Q = 200$ litres/s $\Rightarrow 0.2 \text{ m}^3/\text{s}$

Head lost due to friction, $h_f = 4$ m

Value of Chezy's constant, $C = 50$

Find:

Diameter of the pipe, $d = ?$

Sol:

velocity of flow

$$V = \frac{\text{Discharge}}{\text{Area}}$$

$$= \frac{Q}{\pi/4 \, d^2} = \frac{0.2}{\pi/4 \times d^2}$$

$$V = \frac{0.2 \times 4}{\pi d^2}$$

Hydraulic mean depth, $m = \frac{d}{4}$

Loss of head per unit length, $i = \frac{h_f}{L}$

$= \frac{4}{2000}$

$i = 0.002$

Chezy's formulae,

$$V = C\sqrt{mi}$$

$$\frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times 0.002}$$

$$\frac{0.2 \times 4}{\pi d^2 \times 50} = \sqrt{\frac{d}{4} \times 0.002}$$

$$\frac{0.00509}{d^2} = \sqrt{\frac{d}{4} \times 0.002}$$

Squaring both sides,

$$\frac{(0.00509)^2}{d^4} = \frac{d}{4} \times 0.002$$

$$\frac{0.0000259}{d^4} = \frac{d}{4} \times 0.002$$

$$d^5 = \frac{0.0000259 \times 4}{0.002}$$

$$d^5 = 0.0518$$

$$d = (0.518)^{1/5}$$

$$d = 0.553\ m$$

Diameter of Pipe, $\boxed{d = 553\ mm}$

3) A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300 mm at the rate of 300 litres/sec. Find the head lost due to friction for a length of 50 m of the pipe.

Data:

Kinematic viscosity, $\nu = 0.4$ stoke $\Rightarrow 0.4 cm^2/s$

$\nu \Rightarrow 0.4 \times 10^{-4} m^2/s$

Diameter of pipe, $d = 300$ mm $\Rightarrow 0.30$ m

Discharge, $Q = 300$ litres/sec $\Rightarrow 0.3 m^3/s$

Length of pipe, $L = 50$ m

Find:

Head lost due to friction $* h_f = ?$

Sol:

velocity of flow,

$$V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{A}$$

$$= \frac{Q}{\frac{\pi}{4} d^2}$$

$$= \frac{Q \times 4}{\pi \times d^2} = \frac{0.3 \times 4}{\pi \times 0.3^2}$$

$$V = 4.24 \text{ m/s}$$

$\therefore$ Reynolds Number, $Re = \frac{V \times d}{\nu}$

$$= \frac{4.24 \times 0.3}{0.4 \times 10^{-4}}$$

$$Re = 3.18 \times 10^4$$

Re is more than 4000, so the formulae f is given by,

$$f = \frac{0.0079}{Re^{1/4}}$$

$$= \frac{0.079}{(3.18 \times 10^4)^{1/4}}$$

$$f = 0.00591$$

∴ Head lost due to friction,

$$h_f = \frac{4 \cdot f \cdot L \cdot v^2}{2gd}$$

$$= \frac{4 \times 0.00591 \times 50 \times (4.27)^2}{2 \times 9.81 \times 0.3}$$

$$\boxed{h_f = 3.61 \text{ m}}$$

---

4) An oil of specific gravity 0.7 is flowing through a pipe of diameter 30 cm at the rate of 500 litres/sec. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m. Take kinematic viscosity of oil 0.29 stokes.

Data:

$S = 0.7$

$d = 30 \text{ cm} \Rightarrow 300 \text{ mm} \Rightarrow 0.3 \text{ m}$

$Q = 500 \text{ lit/sec} \Rightarrow 0.5 \text{ m}^3/\text{s}$

$\nu = 0.29 \text{ stokes} \Rightarrow 0.29 \times 10^{-4} \text{ m}^2/\text{s}$

$L = 1000 \text{ m}$

Find :

i) Heat lost due to friction, h_f

ii) Power, P

Sol :

i) Heat lost due to friction, h_f

$$h_f = \frac{4 \times f \times L \times V^2}{2 \times g \times d}$$

$$Q = A \times V$$

$$V = \frac{Q}{Area}$$

$$= \frac{0.5}{\pi/4 \times (0.3)^2}$$

$$V = 7.07 \text{ m/s}$$

$$Re = \frac{V.d}{\nu}$$

$$= \frac{7.07 \times 0.3}{0.29 \times 10^{-4}}$$

$$Re = 73137.93$$

Re is more than 4000

$$f = \frac{0.079}{Re^{1/4}}$$

$$= \frac{0.079}{(73137.93)^{1/4}}$$

$$f = 0.0048$$

∴ Heat lost due to friction,

$$h_f = \frac{4fLV^2}{2gd}$$

$$= \frac{4 \times 0.0048 \times 1000 \times (7.07)^2}{2 \times 9.81 \times 0.3}$$

$$h_f = 163.04 \text{ m}$$

ii) Power required,

$$P = \frac{\rho \cdot g \cdot Q \cdot h_f}{1000}$$

Specific Gravity, $S = \frac{\rho_{oil}}{\rho_{water}}$

$$\rho_{oil} = S \times \rho_{water}$$

$$= 0.7 \times 1000$$

$$\rho_{oil} = 700 \text{ kg/m}^3$$

∴ Power, $P = \frac{700 \times 9.81 \times 0.5 \times 163.04}{1000}$

$$P = 559.79 \text{ kW}$$

P.NO: 581

Reynold's Number:

It is defined as the ratio of inertia force of a flowing fluid & the viscous force of the fluid.

$$Re = \frac{V \times d}{\nu}$$

(or)

$$Re = \frac{\rho V d}{\mu}$$

P.NO: 410

5) An oil of specific Gravity 0.9 & viscosity 0.06 poise is flowing through a pipe of diameter 200 mm at the rate of 60 lit/sec. Find the head lost due to friction for a 500m length of pipe. Find the power required to maintain this flow.

Data:

Sp. gr of oil = 0.9

viscosity, $\mu = 0.06$ poise $= \frac{0.06}{10}$ Ns/m^2

Dia. of pipe, $d = 200$ mm $\Rightarrow 0.2$m

Discharge, $Q = 60$ lit/s $\Rightarrow 0.06$ m^3/s

Length, $L = 500$ m

Density, $\rho = 0.9 \times 1000 \Rightarrow 900$ kg/m^3

Find:

Power required,

Sol:

Reynolds number $Re = \frac{\rho V d}{\mu} = 900 \times \frac{V \times 0.2}{0.06/10}$

$$V = \frac{Q}{A} = \frac{0.06}{\pi/4 (0.2)^2} = 1.909 \text{ m/s}$$

$$Re = \frac{900 \times 1.91 \times 0.2 \times 10}{0.06} = 57300$$

If Re lies between 4000 and 10^5, the value of co-efficient of friction, f

$$f = \frac{0.079}{Re^{0.25}} = \frac{0.079}{(57300)^{0.25}} = 0.0051$$

∴ Head lost due to friction,

$$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$$

$$= \frac{4 \times 0.0051 \times 500 \times 1.91^2}{0.2 \times 2 \times 9.81}$$

$$h_f = 9.48 \text{ m}$$

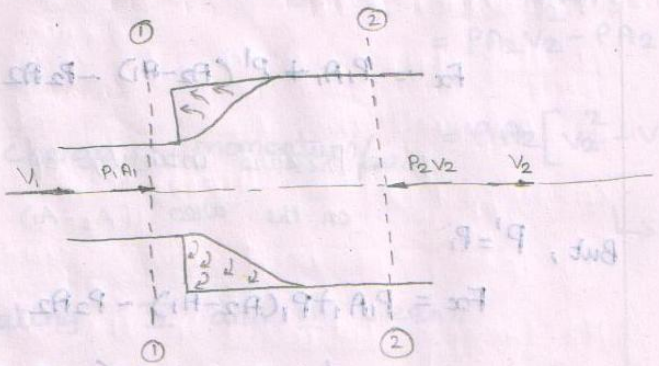
∴ Power required,

$$P = \frac{\rho g . Q . h_f}{1000}$$

$$= \frac{900 \times 9.81 \times 0.06 \times 9.48}{1000}$$

$$P = 5.02 \text{ kw}$$

Loss of Head Due to sudden Enlargement:



Let,

P_1 ⇒ Pressure intensity at section 1-1,

V_1 ⇒ Velocity of flow at section 1-1,

A_1 ⇒ Area of pipe at section 1-1,

P_2 ⇒ Pressure intensity at section 2-2,

V_2 ⇒ Velocity of flow at section 2-2,

A_2 ⇒ Area of pipe at section 2-2.

Applying Bernoulli's equation at section 1-1 & 2-2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + he$$

he ⇒ loss of head due to sudden Enlargement

But, pipe is horizontal, $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + he$$

$$he = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \longrightarrow I\text{-eqn}$$

Consider the control volume of liquid between sections 1-1 and 2-2.

$$F_x = P_1A_1 + P'(A_2 - A_1) - P_2A_2$$

$P' \Rightarrow$ Pressure intensity of the liquid eddies on the area $(A_2 - A_1)$

But, $P' = P_1$

$$F_x = P_1A_1 + P_1(A_2 - A_1) - P_2A_2$$

$$= P_1A_1 + P_1A_2 - P_1A_1 - P_2A_2$$

$$F_x = P_1A_2 - P_2A_2$$

$$F_x = A_2(P_1 - P_2) \longrightarrow \text{II-eqn}$$

Momentum of liquid/sec. at section 1-1,

$$= \text{mass} \times \text{velocity}$$

$$= \rho A_1V_1 \times V_1$$

$$= \rho A_1V_1^2$$

momentum of liquid/sec at section 2-2.

$$= \rho A_2V_2 \times V_2$$

$$= \rho A_2V_2^2$$

$\therefore$ change of momentum $= \rho A_2V_2^2 - \rho A_1V_1^2$

But, From continuity equation,

$$A_1V_1 = A_2V_2$$

$$A_1 = \frac{A_2V_2}{V_1}$$

∴ change of momentum/sec $= \rho A_2 V_2^2 - \rho \times \frac{A_2 V_2}{V_1} \times V_1$

$= \rho A_2 V_2^2 - \rho A_2 V_1 V_2$

change of momentum/sec $= \rho A_2 [V_2^2 - V_1 V_2]$

↳ III - eqn

Equating II and III - eqn.

$A_2 (P_1 - P_2) = \rho A_2 [V_2^2 - V_1 V_2]$

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_1 V_2$$

Dividing by g on both sides,

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}$$

sub. the value of $\frac{P_1}{\rho g} - \frac{P_2}{\rho g}$ in eqn - I

$$h_e = \frac{V_2^2 - V_1 V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$= \frac{2V^2 - 2V_1 V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_2^2 - V_1^2 - 2V_1 V_2}{2g}$$

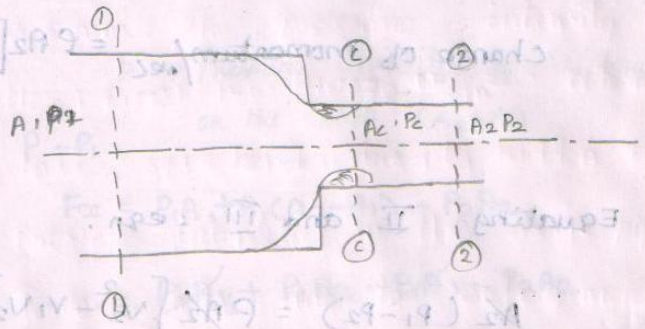
$$= \left[\frac{V_1 - V_2}{2g}\right]^2$$

$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

∴ This loss of head due sudden Enlargement is verified.

P.NO: 473

Loss of Head due to sudden construction:-



Let,

$A_c \Rightarrow$ Area of velocity at section c-c

$V_c \Rightarrow$ velocity of flow at section c-c

$A_2 \Rightarrow$ Area of flow at section 2-2

$V_2 \Rightarrow$ velocity of flow at section 2-2

$A_1 \Rightarrow$ Area of flow at section 1-1

$V_1 \Rightarrow$ velocity of flow at section 1-1.

$h_c \Rightarrow$ Loss of head due to sudden construction

From sudden Enlargement,

$$= \frac{(V_c - V_2)^2}{2g}$$

$$= \frac{1}{2g}\left(V_c \times \frac{V_2}{V_2} - V_2 \times \frac{V_2}{V_2}\right)^2$$

$$= \frac{1}{2g}\left[V_2\left(\frac{V_c}{V_2} - 1\right)\right]^2$$

$$= \frac{1}{2g} V_2^2 \left[\frac{V_c}{V_2} - 1\right]^2$$

$$= \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2 \longrightarrow \text{I-eqn}$$

From continuity Equation,

$$A_c V_c = A_2 V_2$$

$$[or] \quad \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{A_c/A_2} = \frac{1}{C_c}$$

$$\left[\because C_c = \frac{A_c}{A_2} \right]$$

substituting the value $\frac{V_c}{V_2}$

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$h_c = \frac{K V_2^2}{2g}$$

where, $K \Rightarrow \left[\frac{1}{C_c} - 1\right]^2$

If the value of C_c is assumed to be equal to 0.62, then

$$K = \left[\frac{1}{0.62} - 1\right]^2$$

$$K = 0.375$$

Then, h_c becomes,

$$h_c = \frac{K V_2^2}{2g}$$

$$= 0.375 \times \frac{V_2^2}{2g}$$

The value of C_c is not given then the head loss due to contraction is taken as,

$$h_c = 0.5 \frac{V_2}{2g}$$

P.NO: 475 (X)

The rate of flow of water through a horizontal pipe is $0.25\ m^3/s$. The diameter of the pipe which is 200 mm suddenly enlarged to 400 mm. The pressure intensity in the smaller pipe is $11.772\ N/cm^2$. Determine

i) loss of head due to sudden Enlargement
ii) Pressure intensity in the large pipe
iii) Power lost due to enlargement.

Data:

Discharge, $Q = 0.25\ m^3/s$

$d_1 = 200\ mm \Rightarrow 0.2\ m$

$d_2 = 400\ mm \Rightarrow 0.4\ m$

Pressure Intensity in the smaller pipe $= 11.772\ N/cm^2$

$\Rightarrow 11.772 \times 10^4\ N/m^2$

Find:

i) loss of head due to sudden Enlargement
ii) Pressure intensity in the large pipe
iii) Power lost due to enlargement.

Sol:

Area, A_1

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi \times (0.2)^2}{4} = 0.03141\ m^2$$

Area, A_2

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi \times (0.4)^2}{4} = 0.125\ m^2$$

From continuity equation,

$$Q = A_1V_1 = A_2V_2$$

$$Q = A_1V_1$$

$$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = 7.96 \text{ m/s}$$

$$Q = A_2V_2$$

$$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.125} = 1.99 \text{ m/s}$$

i) loss of head due to sudden Enlargement

$$he = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(7.96 - 1.99)^2}{2 \times 9.81}$$

$$\boxed{he = 1.816 \text{ m}}$$

ii) Pressure Intensity in the large pipe,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + he$$

For horizontal pipe,

$$Z_1 = Z_2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + he$$

$$\frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - he$$

$$= \frac{11.792 \times 10^4}{1000 \times 9.81} + \frac{(7.96)^2}{2 \times 9.81} - \frac{(1.99)^2}{2 \times 9.81} - 1.816$$

$$= 12.0 + 3.229 - 0.2018 - 1.816$$

$$\frac{P_2}{\rho g} = 15.229 - 2.0178$$

$$\frac{P_2}{\rho g} = 13.21 \text{ m of water}$$

$$P_2 = 13.21 \times \rho \times g$$

$$= 13.21 \times 1000 \times 9.81$$

$$\boxed{P_2 = 12.96 \text{ N/cm}^2}$$

iii) Power lost due to sudden Enlargement,

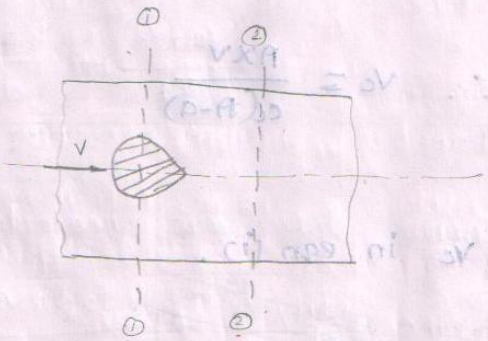
$$P = \frac{\rho g \cdot Q \cdot he}{1000}$$

$$= \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000}$$

$$\boxed{P = 4.453 \text{ kW}}$$

P.NO, 482

Loss of Head due to an obstruction in a Pipe:



Let,

a → Maximum area of obstruction

V → velocity of liquid pipe

A → Area of Pipe

Loss of Head due to obstruction } = Loss of head due to enlargement from vena-contracta to section 2-2

$$= \frac{(V_c - V)^2}{2g} \rightarrow (i)$$

From continuity equation, → (ii)

$$a_c \times V_c = A \times V$$

where,

a_c = area of cross section at vena-contracta

C_c = co-efficient of construction

$$C_c = \frac{\text{area at vena-contracta}}{(A - a)} = \frac{a_c}{(A-a)}$$

$$a_c = C_c \times (A - a)$$

∴ substituting this value (ii)

$$C_c \times (A - a) \times V_c = A \times V$$

$$\therefore \quad V_c = \frac{A \times V}{C_c (A - a)}$$

sub V_c in eqn (i).

$$\text{Head Loss due to obstruction} = \frac{(V_c - V)^2}{2g}$$

$$= \frac{\left(\frac{A \times V}{C_c (A - a)} - V\right)^2}{2g}$$

$$\text{Head loss due to obstruction} = \frac{V^2}{2g} \left[\frac{A}{C_c (A - a)} - 1\right]^2$$

Loss of Head due to Bend in Pipe:

$$h_b = \frac{KV^2}{2g}$$

K → Angle of bend (or) Radius of curvature of bend

Loss of Head in various pipe fittings:

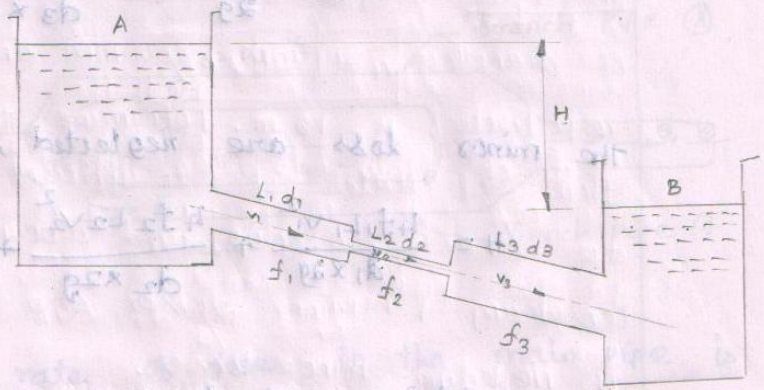
$$\text{Loss of Head in various pipe fittings} = \frac{KV^2}{2g}$$

where,

V ⇒ velocity of flow

K ⇒ co-efficient of pipe fitting

Flow Through pipes in series (or) Flow through compound pipes:



Let,

L_1, L_2 & L_3 → Length of pipes 1, 2 & 3 respectively.

d_1, d_2 & d_3 → Diameter of pipes 1, 2, & 3 respectively,

V_1, V_2, V_3 → velocity of flow through pipes 1, 2 & 3.

f_1, f_2 & f_3 → co-efficient of frictions for pipes 1, 2 & 3

H → Difference of water level in two tanks

The discharge passing through each pipe is same,

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$H = \frac{0.5 v_1^2}{2g} + \frac{4 f_1 L_1 v_1^2}{d_1 \times 2g} + \frac{0.5 v_2^2}{2g} + \frac{4 f_2 L_2 v_2^2}{d_2 \times 2g} + \frac{(v_2 - v_3)^2}{2g} + \frac{4 f_3 L_3 v_3^2}{d_3 \times 2g} + \frac{v_3^2}{2g}$$

The minor loss are neglected,

$$H = \frac{4 f_1 L_1 v_1^2}{d_1 \times 2g} + \frac{4 f_2 L_2 v_2^2}{d_2 \times 2g} + \frac{4 f_3 L_3 v_3^2}{d_3 \times 2g}$$

The co-efficient of friction is same for all pipes.

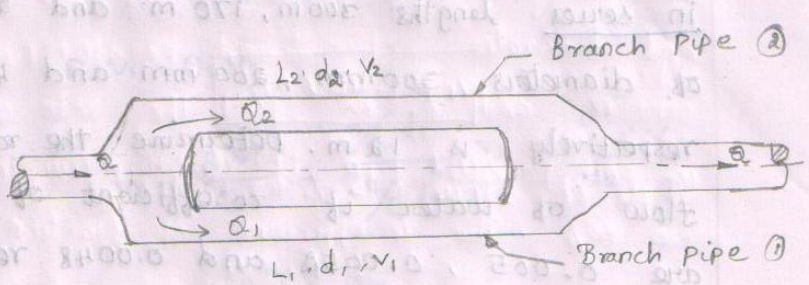
$$f_1 = f_2 = f_3 = f$$

$$\therefore H = \frac{4 f L_1 v_1^2}{d_1 \times 2g} + \frac{4 f L_2 v_2^2}{d_2 \times 2g} + \frac{4 f L_3 v_3^2}{d_3 \times 2g}$$

$$H = \frac{4f}{2g} \left[\frac{L_1 v_1^2}{d_1} + \frac{L_2 v_2^2}{d_2} + \frac{L_3 v_3^2}{d_3} \right]$$

P.NO: 508

Flow through parallel pipes:



The rate of flow in the main pipe is equal to the sum of the rate of flow through branch pipes.

$$Q = Q_1 + Q_2$$

The loss of head for each branch pipe is same.

Loss of Head for Branch pipe 1 = loss of Head for Branch pipe 2

$$\frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g}$$

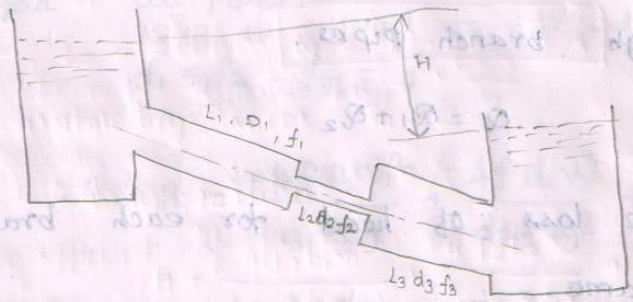
$\therefore f_1 = f_2$

$$\therefore \boxed{\frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{d_2 \times 2g}}$$

P.NO: 503

Problems:

The difference in water surface levels in two tanks, which are connected by three pipes in series lengths 300 m, 170 m and 210 m and of diameters 300 mm, 200 mm and 400 mm respectively, is 12 m. Determine the rate of flow of water if co-efficient of friction are 0.005, 0.0052 and 0.0048 respectively. Considering i) Minor losses also
ii) Neglecting minor losses.



Data:

$L_1 = 300$ m

$L_2 = 170$ m

$L_3 = 210$ m

Diameter,

$d_1 = 300\text{mm} \Rightarrow 0.3\text{ m}$

$d_2 = 200\text{mm} \Rightarrow 0.2\text{m}$

$d_3 = 400\text{mm} \Rightarrow \frac{400}{1000} \Rightarrow 0.4\text{m}$

Friction,

$f_1 = 0.005$

$f_2 = 0.0052$

$f_3 = 0.0048$

$H = 12$ m

Find:

i) Minor losses also ii) Neglecting minor losses

Sol: i) Minor losses also

$Q = A_1V_1 = A_2V_2 = A_3V_3$

$A_1V_1 = A_2V_2$

PNO: 478

$$V_2 = \frac{A_1V_1}{A_2} = \frac{\pi/4\, d_1^2}{\pi/4\, d_2^2} V_1 = \frac{d_1^2}{d_2^2} V_1$$

$$= \left(\frac{0.3}{0.2}\right)^2 \times V_1$$

$$\underline{V_2 = 2.25 V_1}$$

$$V_3 = \frac{A_1V_1}{A_3} = \frac{d_1^2}{d_3^2} = \left(\frac{0.3}{0.4}\right)^2 \times V_1$$

$$\underline{V_3 = 0.5625 V_1}$$

Now using equation,

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g}$$

Substituting V_2 and V_3

$$12.0 = \frac{0.5V_1^2}{2g} + \frac{4 \times 0.005 \times 300 \times V_1^2}{0.3 \times 2g} + \frac{0.5 \times (2.25V_1)^2}{2g}$$

$$+ \frac{4 \times 0.0052 \times 170 \times (2.25V_1)^2}{0.2 \times 2g} + \frac{(2.25V_1 - 0.5625V_1)^2}{2g}$$

$$+ \frac{4 \times 0.0048 \times 210 \times (0.5625V_1)^2}{0.4 \times 2g} + \frac{(0.5625V_1)^2}{2g}$$

$$12.0 = \frac{V_1^2}{2g}\left[0.5 + 20.0 + 2.53 + 89.505 + 2.847 + 3.189 + 0.316\right]$$

$$12.0 = \frac{V_1^2}{29} \quad [118.887]$$

$$V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{118.887}}$$

$$\boxed{V_1 = 1.407 \text{ m/s}}$$

Rate of flow, Q = Area × velocity

$$= A_1 \times V_1$$

$$Q = \pi/4 \; d_1^2 \times V_1$$

$$= \pi/4 \times (0.3)^2 \times 1.407$$

$$\boxed{Q = 99.45 \text{ lit/s}}$$

(ii) Neglecting minor losses,

using equation,

$$H = \frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{4 f_3 L_3 V_3^2}{d_3 \times 2g}$$

$$12.0 = \frac{V_1^2}{2g} \left[\frac{4 \times 0.005 \times 300}{0.3} + \frac{4 \times 0.0052 \times 170 \times (2.25)^2}{0.2} + \frac{4 \times 0.0048 \times 210 \times (0.5625)^2}{0.4} \right]$$

$$12.0 = \frac{V_1^2}{2g} \left[20.0 + 89.505 + 3.189 \right]$$

$$12.0 = \frac{V_1^2}{2g} \times 112.694$$

$$V_1 = \sqrt{\frac{2 \times 9.81 \times 12.0}{112.694}}$$

$$\boxed{V_1 = 1.445 \text{ m/s}}$$

Discharge $Q = A_1 \times V_1 = \pi/4 \times (0.3)^2 \times 1.445$

$$\boxed{Q = 102.1 \text{ lit/s}}$$

Three pipes of lengths 800 m, 500 m and 400 m and of diameters 500 mm, 400 mm and 300 mm respectively are connected in series. These pipes are to be replaced by a single pipe of length 1700 m. Find diameter of the single pipe.

Data:

Lengths,

$L_1 = 800\,m$

$L_2 = 500\,m$

$L_3 = 400\,m$

Diameters,

$d_1 = 500\,mm \Rightarrow 0.5\,m$

$d_2 = 400\,mm \Rightarrow 0.4\,m$

$d_3 = 300\,mm \Rightarrow 0.3\,m$

Single pipe length, $L = 1700\,m$

Find:

Diameter of single pipe, $d = ?$

Sol:

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\frac{1700}{d^5} = \frac{800}{0.5^5} + \frac{500}{0.4^5} + \frac{400}{0.3^5}$$

$$\frac{1700}{d^5} = 25600 + 48828.125 + 164609$$

$$\frac{1700}{d^5} = 239037$$

$$d^5 \times 239037 = 1700$$

$$d^5 = \frac{1700}{239037} = 0.007118$$

$$d = 0.3718\,m \Rightarrow \boxed{d = 371.8\,mm}$$

P. No. 509

Pr. No - 11.32

A main pipes divides into two parallel pipes which again forms one pipe as shown in fig. The length & diameter for first parallel pipe are 2000 m and 1.0 m respectively. while the length and diameter of second parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe if total flow in main is 3.0 m^3/s. The co-efficient of friction for each parallel pipe is same & equal to 0.005.

Data:

$L_1 = 2000$ m

$d_1 = 1.0$ m

$L_2 = 2000$ m

$d_2 = 0.8$ m

Discharge $Q = 3.0\ m^3/s$

Co-efficient of friction $f_1 = f_2 = f = 0.005$.

Find:

~~$Q = Q_1 + Q_2 = ?$~~

i) $Q_1 = ?$

ii) $Q_2 = ?$

Sol:

$$\frac{4 f L_1 V_1^2}{2g d_1} = \frac{4 f L_2 V_2^2}{2g d_2}$$

$$\frac{4 \times 0.005 \times 2000 \times V_1^2}{2 \times 9.81 \times 1.0} = \frac{4 \times 0.005 \times 2000 \times V_2^2}{2 \times 9.81 \times 0.8}$$

$$\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8}$$

$$V_1^2 = \frac{V_2^2}{0.8} \times 1.0$$

$$V_1^2 = \frac{V_2^2}{0.8}$$

$$V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{0.894}$$

$$Q_1 = \pi/4\, d_1^2 \times V_1 = \pi/4\,(1)^2 \times \frac{V_2}{.894}$$

$$Q_2 = \pi/4\, d_2^2 \times V_2 = \pi/4\,(0.8)^2 \times V_2 = \pi/4 \times 0.64 \times V_2$$

Sub Q_1 & Q_2 value in I-eqn

$$\pi/4 \times \frac{V_2}{0.894} + \pi/4 \times 0.64\, V_2 = 3.0$$

$$0.8785\, V_2 + 0.5026\, V_2 = 3.0$$

$$V_2 \left[0.8785 + 0.5026\right] = 3.0$$

$$V_2 \times 1.3811 = 3.0$$

$$V_2 = \frac{3.0}{1.3811}$$

$$V_2 = \underline{2.17\ m/s}$$

$$V_1 = \frac{V_2}{0.894} = \frac{2.17}{0.894} = \underline{2.427\ m/s}$$

Hence,

$$Q_1 = \pi/4\, d_1^2 \times V_1$$

$$= \pi/4\, 1^2 \times 2.427$$

$$\boxed{Q_1 = 1.906\ m^3/s}$$

$$Q_2 = Q - Q_1$$

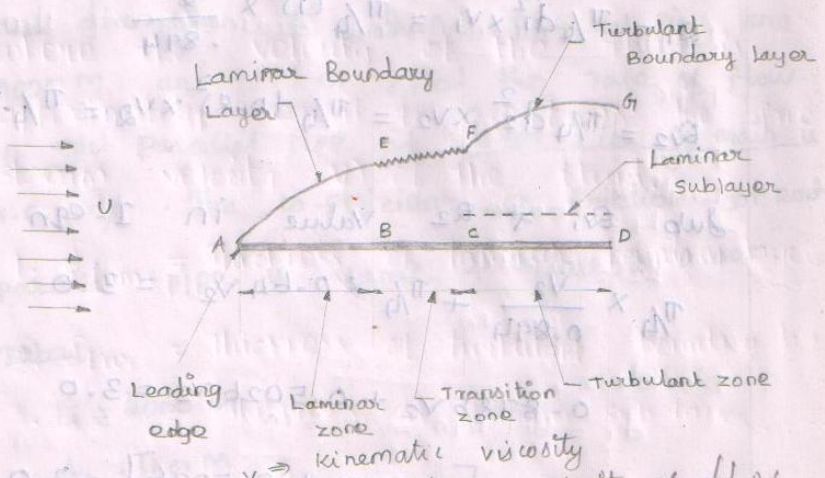
$$= 3.0 - 1.906$$

$$\boxed{Q_2 = 1.094\ m^3/s}$$

Pno-611

Boundary Layer Flow:

A very thin layer of the fluid, is called the "boundary layer".



$\nu \Rightarrow$ kinematic viscosity

$U \Rightarrow$ Free Steam velocity of fluid

Reynolds Number $(Re)_x = \frac{Ux \times x}{\nu}$

(*) Moody Diagram:

Reference

Moody diagrams present various values of friction factor (f) Reynolds number Re and relative roughness.

[or]

Moody diagram gives the value of Friction factor of any pipe with its relative roughness (κ/D) and Reynolds number of flow Re known,

κ = Absolute roughness

D = Diameter of pipe

Hence Moody's diagram is the most common source of reference for obtaining value of f.

$Re \propto$

P.NO: 613

Turbulant Boundary Layer:

If the length of the plate is more than the distance x, the thickness of boundary layer will go on increasing in the down-stream direction. Then the laminar boundary layer becomes unstable & motion of fluid within it, is distributed and irregular which leads to a transition from laminar to boundary layer. The short length over which the boundary layer flow changes from laminar to turbulant is called "transition zone". Further downstream the transition zone, the boundary layer is turbulant & continuous to grow in thickness. This layer of boundary is called "turbulant boundary layer".

Laminar sub-layer:

The thickness of boundary sub-layer is go very small. $\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = \mu \frac{u}{y}$

Laminar Boundary Layer:

Near the leading edge of the surface of the plate, when the thickness is small, the flow in the boundary layer is laminar through the main flow is turbulant. This layer of the fluid is said to be laminar boundary layer.

$$(Re)_x = \frac{U \times x}{\nu}$$

x → Distance from leading edge

ν – kinematic viscosity

U – Free steam velocity of fluid

P.NO - 613

Boundary Layer Thickness :

It is defined as the distance from boundary of the solid body measured in the y-direction to the point, where the velocity of the fluid is approximately equal to 0.99 times the free stream velocity (V) of the fluid.

1. $\delta_{lam.}$ = Thickness of laminar boundary layer
2. $\delta_{tur.}$ = Thickness of turbulant boundary layer
3. δ' = Thickness of laminar sub layer.

Displacement Thickness (δ^*) :

It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation.

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{v}\right) . dy$$