

MA8402 – PROBABILITY AND QUEUEING THEORY

(REGULATION 2017)

Prepared by,

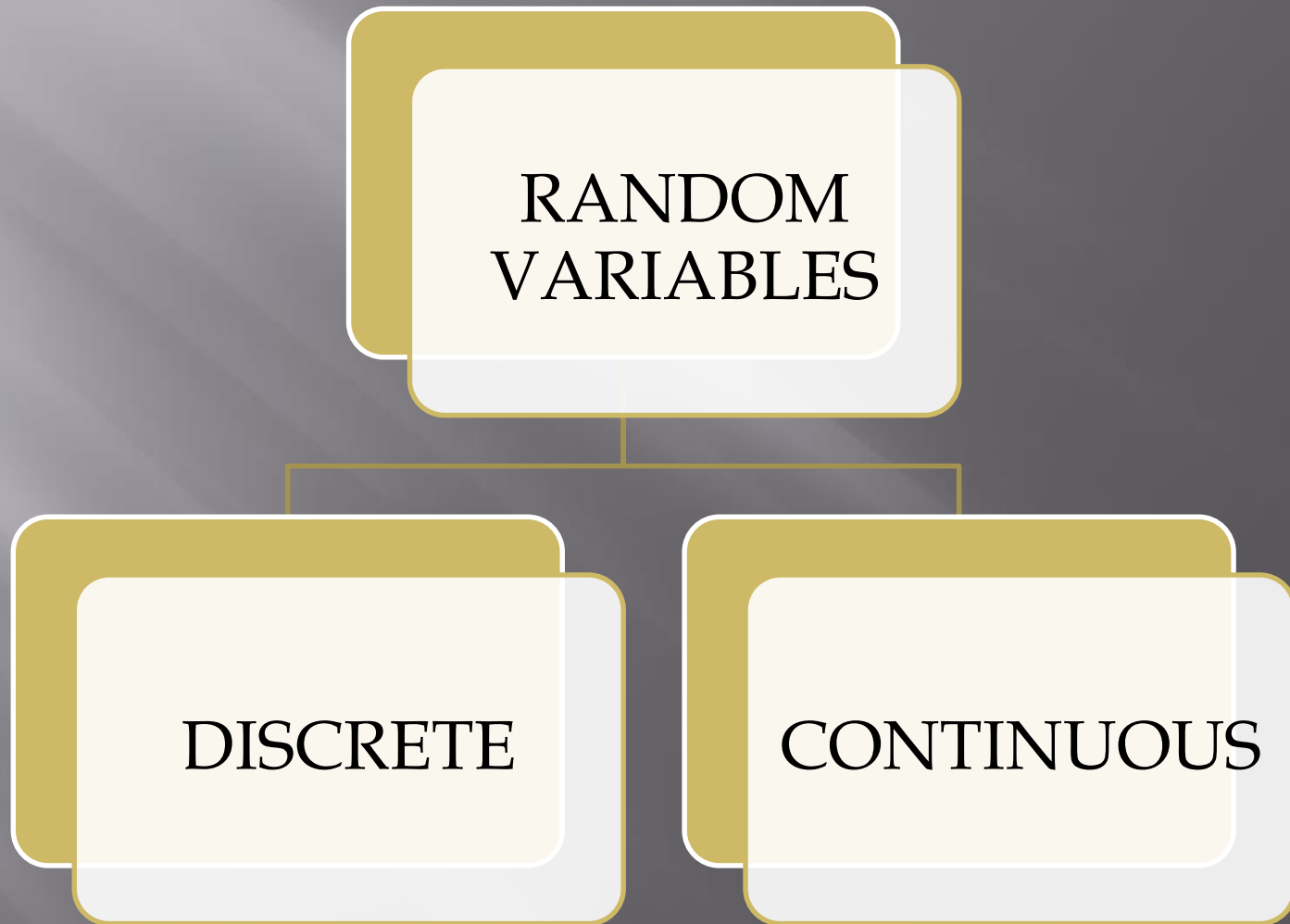
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UNIT-2

TWO DIMENSIONAL RANDOM VARIABLES

- Many situations of our Engineering problems are handled by the theory of two random variables.
- Hence such important concepts as auto correlation, cross – correlation and co-variance functions, which apply to random processes , are based on two random variables

RANDOM VARIABLES



Marginal Probability Function of X

- ▣ If the joint probability distribution of two random variables X and Y is given then the marginal probability function of X is given by
- ▣ $P_x(x_i) = p_i$ (marginal probability function of Y)
- ▣
- ▣ **Conditional Probabilities**
- ▣ The conditional probabilities function of X given $Y = y_j$ is given by $P(X/Y)$

Example 1

From the following joint distribution of X and Y find the marginal distributions.

| $X \backslash Y$ | 0 | 1 | 2 |
|------------------|----------------|----------------|----------------|
| 0 | $\frac{3}{28}$ | $\frac{9}{28}$ | $\frac{3}{28}$ |
| 1 | $\frac{3}{14}$ | $\frac{3}{14}$ | 0 |
| 2 | $\frac{1}{28}$ | 0 | 0 |

Solution

Solution

| | | | |
|-------------------|--------------------------------------------|----------------------------|--------------------------|
| X \ Y | 0 | 2 | $P_Y(y) = p(Y=y)$ |
| 0 | $\frac{3}{28} P(0,0)$ | $\frac{3}{28} P(2,0)$ | $\frac{15}{28} = P_Y(0)$ |
| 1 | $\frac{3}{14} P(0, 1)$ | $\frac{3}{14} P(1,1)$ | $\frac{6}{14} = P_Y(1)$ |
| 2 | $\frac{1}{28} P(0,2)$ | $0 P(2,2)$ | $\frac{1}{28} = P_Y(2)$ |
| $P_X(X) = P(X=x)$ | $\frac{10}{28} = \frac{5}{14}$ $P_X(0)$ | $\frac{3}{28}$ $P_X(2)$ | 1 |

The marginal distribution of X

$$P_X(0) = P(X = 0) = p(0,0) + p(0,1) + p(0,2) = \frac{5}{14}$$

$$P_X(1) = P(X = 1) = p(1,0) + p(1,1) + p(1,2) = \frac{15}{28}$$

$$P_X(2) = P(X = 2) = p(2,0) + p(2,1) + p(2,2) = \frac{3}{28}$$

The marginal distribution of X

$$P_X(0) = P(X = 0) = p(0,0) + p(0,1) + p(0,2) = 5/14$$

$$P_X(1) = P(X = 1) = p(1,0) + p(1,1) + p(1,2) = 15/28$$

$$P_X(2) = P(X = 2) = p(2,0) + p(2,1) + p(2,2) = 3/28$$

Marginal probability function of X is

$$P_X(x) = \begin{cases} \frac{5}{14}, & x = 0 \\ \frac{15}{28}, & x = 1 \\ \frac{3}{28}, & x = 2 \end{cases}$$

The marginal distribution of Y

$$P_Y(0) = P(Y = 0) = p(0,0) + p(1,0) + p(2,0) = 15/28$$

$$P_Y(1) = P(Y = 1) = p(0,1) + p(2,1) + p(1,1) = 3/7$$

$$P_Y(2) = P(Y = 2) = p(0,2) + p(1,2) + p(2,2) = 1/28$$

Marginal probability function of Y is

$$P_Y(y) = \begin{cases} \frac{15}{28}, & y = 0 \\ \frac{3}{7}, & y = 1 \\ \frac{1}{28}, & y = 2 \end{cases}$$

THANK YOU